

Hausaufgabenserie 8

Adrian Schollmeyer

Aufgabe 1

| \circ | a_1 | a_2 | a_3 | a_4 | a_5 | a_6 |
|---------|-------|-------|-------|-------|-------|-------|
| a_1 | a_1 | a_2 | a_3 | a_4 | a_5 | a_6 |
| a_2 | a_2 | a_1 | a_5 | a_6 | a_3 | a_4 |
| a_3 | a_3 | a_6 | a_1 | a_5 | a_4 | a_2 |
| a_4 | a_4 | a_5 | a_6 | a_1 | a_2 | a_3 |
| a_5 | a_5 | a_4 | a_2 | a_3 | a_6 | a_1 |
| a_6 | a_6 | a_3 | a_4 | a_2 | a_1 | a_5 |

$$e_{S_3} = a_1 \tag{1}$$

$$a_1^{-1} = a_1 \tag{2}$$

$$a_2^{-1} = a_2 \tag{3}$$

$$a_3^{-1} = a_3 \tag{4}$$

$$a_4^{-1} = a_4 \tag{5}$$

$$a_5^{-1} = a_6 \tag{6}$$

$$a_6^{-1} = a_5 \tag{7}$$

Aufgabe 2

(a)

$$a_5 \circ a_3 = a_2 \tag{1}$$

$$a_2 \circ x \circ a_2 = a_4 \tag{2}$$

$$\implies x \circ a_2 = a_6 \tag{3}$$

$$\implies x = a_3 \tag{4}$$

(b)

$$x^2 \circ a_2 = a_4 \quad (5)$$

$$x \circ x \circ a_2 = a_4 \quad (6)$$

$$y \circ a_2 = a_4 \quad (7)$$

$$\implies y = a_5 \quad (8)$$

$$y = x \circ x \quad (9)$$

$$\implies x = a_6 \quad (10)$$

Aufgabe 3

(a)

$$2^{2017} \equiv (2^{16})^{64} \cdot 2^{993} \pmod{17} \quad (1)$$

$$2^{2017} \equiv (2^{16})^{32} \cdot 2^{481} \pmod{17} \quad (2)$$

$$2^{2017} \equiv (2^{16})^{16} \cdot 2^{225} \pmod{17} \quad (3)$$

$$2^{2017} \equiv (2^{16})^8 \cdot 2^{97} \pmod{17} \quad (4)$$

$$2^{2017} \equiv (2^{16})^4 \cdot 2^{33} \pmod{17} \quad (5)$$

$$2^{2017} \equiv (2^{16})^2 \cdot 2^1 \pmod{17} \quad (6)$$

$$2^{2017} \equiv 2 \pmod{17} \quad (7)$$

$$\implies 2^{2017} \pmod{17} = 2 \quad (8)$$

(b)

$$5^{2017} \equiv 5^2 \cdot 5^{2014} \cdot 5 \pmod{12} \quad (9)$$

$$\equiv 1 \cdot (5^2)^{1007} \cdot 5 \pmod{12} \quad (10)$$

$$\equiv 1^{1007} \cdot 5 \pmod{12} \quad (11)$$

$$\equiv 5 \pmod{12} \quad (12)$$

$$\implies 5^{2017} \pmod{12} = 5 \quad (13)$$